Ch. 8 Modern HW

Problem 2---Given electrons with 26,000eV---what is minimum wavelength x-ray produced?

$$e\Delta V = \frac{hc}{\lambda_{min}}$$

$$\lambda_{min} = \frac{hc}{e\Delta V}$$

$$= 47.8 \text{ pm}$$

8-4 An electron with an initial speed v(i) 1.50x10^{7}m/s producing an x-ray of 4.0nm. What was the final speed?

$$V_i = 1.50 \times 10^7 \text{ m/s} \qquad \gamma_i = 1.00125$$

$$\nu = \frac{c}{\lambda} = 7.5 \times 10^{16} \text{ Hz}$$

$$m_0 c^2 = 0.511 \text{ MeV} = 8.199 \times 10^{-14} \text{ J}$$

Now use conservation of energy---either classical or relativistic (is it moving slowly???)

$$\gamma_f m_0 c^2 = \gamma_i m_0 c^2 - h\nu$$

$$\gamma_f = 1.00064 \longrightarrow V_f = \sqrt{1 - 1/\gamma_f^2}\; c$$

$$= 1.073 \times 10^7 \text{ m/s}$$

Classical -

$$\frac{1}{2} m_0 V_f^2 = \frac{1}{2} m_0 V_i^2 - h\nu$$

$$V_f = 1.0739 \times 10^7 \text{ m/s}$$

Problem 8-5

Work back to find initial speed---given electron final speed 3.00E6m/s, and emission of photon 2.0E17 Hz.

$h = 6.626 \times 10^{-34}$ J-s

$$m_0 = 9.11 \times 10^{-31} \text{ kg}$$

$$\tfrac{1}{2} m v_f^2 = \tfrac{1}{2} m v_0^2 - h\nu$$

$$v_i = 1.732 \times 10^{7} \text{ m/s}$$

Is this sufficient to do problem classically, go back and see last problem to do relativistic.

7 An electron is stopped and emits a photon with frequency 1.00iE20 Hz (this is relativistic ---crunch the number out). What was v(i)?

$$\gamma m_0 c^2 = m_0 c^2 + h\nu$$

$$\gamma = \frac{h\nu + m_0 c^2}{m_0 c^2} = 1.808$$

$$v_i = c\sqrt{1 - 1/\gamma^2} = 2.5 \times 10^{8} \text{ m/s}$$

Rel

8-14 The black hole problem

Photons have energy, hence mass (not at rest, but photons are never at rest). We will consider a point mass (could be any amount of mass), and shoot a photon upwards away from the mass starting at some position. As it rises, the photon must lose potential energy---for photons this does not mean, "slow down" (a photon can't do that). Instead it means change wavelength (increase) or frequency (decrease).

a) Mass of photon.

$$m_{ph} = \frac{h\nu}{c^2} = \frac{h}{\lambda c}$$

b) Conservation of energy---show frequency decreases farther from a mass

M = mass object - planet or black hole

m_{ph_R} = mass of photon at R

$$U_{grav} = -\frac{G M m_{ph_R}}{R}$$

see your intro phys books

Consider the photon at two positions as it rises--- R and infinity.

$$E_{ph_\infty} + U_\infty = E_{ph_R} + U_R$$

Note that the potential energy of the photon has risen to the maximum at infinity (of zero) so the frequency term must be minimum. So we have qualitatively proven the point, but let's solve for frequency at R.

$$h\nu_R = h\nu_\infty + \frac{G M m_{ph,R}}{R}$$

Recall we must write the mass of the photon at R in terms of part a. Then gather like terms.

$$h\nu_R = h\nu_\infty + \frac{h\nu_R}{c^2}\left(\frac{GM}{R}\right)$$

$$h\nu_R = \frac{h\nu_\infty}{1 - GM/Rc^2}$$

As R increases the frequency decreases ….eventually reaching the minimum value at infinity.

We could have used any reference point other than infinity, but that is the typical ref point for potential.

c) Do like part b, but now consider positions RA and RB for earth mass with RA radius of Earth and show that we get back, mgh.

$$h\nu_{RB} - h\nu_{RA} = \frac{hGM}{c^2}\left(\frac{\nu_{RB}}{R_B} - \frac{\nu_{RA}}{R_A}\right)$$

$$h\,\Delta\nu$$

If we are on the surface of the Earth then usually RA and RB are close to each other, so both are approximately R, and also the change in frequency will be small.

$R_B > R_A$

$$\nu_{RA} = \nu_{RB} - \Delta\nu$$

$$\cancel{h}\,\Delta\nu = \frac{\cancel{h}GM}{c^2}(\nu_{RB})\left(\frac{1}{R_B} - \frac{1}{R_A}\right) + \frac{\cancel{h}GM\,\Delta\nu}{c^2\,R_A}$$

$$\Delta\nu\left(1 - \frac{GM}{c^2R_A}\right) = \frac{GM}{c^2}\left(\frac{1}{R_B} - \frac{1}{R_A}\right)\nu_{RB}$$

We can drop the Gm/(c^2 RA) term on the left since it is small compared to 1-----crunch the numbers out for Earth, it is small.   Do it. I did.

$$\Delta\nu = \frac{GM}{c^2}\left(\frac{R_A - R_B}{R_A R_B}\right)\nu_R$$

R — A or B

$$h\Delta\nu = \frac{hGM}{c^2}\left(\frac{-\Delta R}{R^2}\right)\nu_R$$

$$\frac{h\nu}{c^2} = m_{photon}$$

$-(R_B - R_A)$

$$\frac{GM}{R^2} = g$$

$-\Delta R = -\Delta y$

& $\Delta\nu$ is negative

$$-\Delta KE_{photon} = m_{photon}\, g\, \Delta y$$

as it must

8-14 part d) Make a black hole. So for a particle to be "free" it must be able to reach infinity with at least some residual kinetic energy. The cutoff is "zero". So we set up our energy equation for the photon starting at some R, and reaching infinity with zero kinetic energy (the potential energy is already zero at infinity).

$$0 = h\nu_R\left(1 - \frac{GM}{c^2 R}\right)$$

So

$$\text{If} \quad \frac{GM}{c^2 R} > 1 \quad \text{then}$$

Light starting at R for such condition will not every make it to infinity (or beyond). As long as that condition is met, we have a black hole.

e) For the sun with mass of 2E30kg, what must R be.

Use the above formula, $R = 1.48 \times 10^3\ \text{m}$

8-17

An x-ray has a maximum energy (electric potential) of 32kV. What is the maximum momentum of x-rays

$$e\Delta V = 1.6 \times 10^{-19} C \times 32{,}000 \text{ Volts}$$
$$= 5.12 \times 10^{-15} J$$

$$p = \frac{E}{c} = 1.71 \times 10^{-23} \text{ kg m/s}$$

8-25

1MeV photons Compton scatter at 5°. Determine new energy.

I will use the wavelength formula we have developed.

$$\Delta\lambda = \frac{h}{mc}(1 - \cos\theta) \qquad \frac{h}{mc} = 2.426 \text{ pm}$$

$$\lambda_{1MeV} = 1.24 \text{ pm}$$

$$\lambda_{scatt} = 1.24 \text{ pm} + 2.426(1 - \cos 5^\circ)$$
$$= 1.24923 \text{ pm}$$

$$E_{scat} = \frac{hc}{\lambda} = 992.6 \text{ KeV}$$

The missing energy went to the electron. Note that like most collisions, the little mass (here the photon) carries most of the kinetic energy!